Integrals II: Now with 50% more variables!

1. Evaluate

(a) $f(x, y, z) = z^4, 2 \leq x \leq 8, 0 \leq y \leq 5, 0 \leq z \leq 1$

(b) $f(x, y, z) = xz^2, 0 \leq x \leq 2, 1 \leq y \leq 6, 3 \leq z \leq 4$

(c) $f(x, y, z) = xe^{y-2z}, 0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq 1$

(d) $f(x, y, z) = \frac{z}{x}, 1 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 4$

(e) $f(x, y, z) = (x + z)^3, 0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$

2. Evaluate $\int\int\int_W f(x, y, z)dV$ **for the function** f **and region** W **specified. Also sketch** W**.**

(a) $f(x, y, z) = x + y, W : y \leq z \leq x, 0 \leq y \leq x, 0 \leq x \leq 1$

(b) $f(x, y, z) = xyz, W : 0 \leq z \leq 1, 0 \leq y \leq \sqrt{1 - x^2}, 0 \leq x \leq 1$

(c) $f(x, y, z) = z$ **and** W **is the region below the upper hemisphere of the sphere** $x^2 + y^2 + z^2 = 9$ **lying over the unit square** $0 \leq x, y \leq 1$**.**

3. Find the volume of the solid in $\mathbb{R}^3$ **bounded by** $y = x^2, x = y^2, z = x + y + 5$**.**

4. Find the average of $xysin(\pi z)$ **over the cube** $0 \leq x, y, z \leq 1$**.**

5. Find the center mass of the cylinder $x^2 + y^2 = 1$ **for** $0 \leq z \leq 1$ **assuming a mass density of** $\rho(x, y, z) = x^2 + y^2$**.**

Integrals III: This time it's polar!

1. Sketch a picture of D, integrate $f(x, y)$ using polar coordinates

(a) $f(x, y) = \sqrt{x^2 + y^2}$**,** $x^2 + y^2 \leq 2$

(b) $f(x, y) = xy, x \geq 0, y \geq 0$ **and** $x^2 + y^2 \leq 4$

(c) $f(x, y) = y(x^2 + y^2)^{-1}, y \geq 1/2, x^2 + y^2 \leq 1$

(d) $\int_0^3 \int_0^{\sqrt{9-y^2}} \sqrt{x^2 + y^2} dxdy$

2. Find the volume of the wedge-shaped region contained in the cylinder $x^2+y^2 = 9$ and bounded above by the plane $z = x$ and below the xy-plane.

3. Use cylindrical coordinates to compute $\int \int \int_W f(x, y, z)dV$ for the given function and region.

(a) $f(x, y, z) = x^2 + y^2, x^2 + y^2 \leq 9, 0 \leq z \leq 5$

(b) $f(x, y, z) = y, x^2 + y^2 \leq 1, x \geq 0, y \geq 1, 0 \leq z \leq 2$

(c) $f(x, y, z) = z, 0 \leq z \leq x^2 + y^2 \leq 9$

4. Use spherical coordinates to calculate the triple integral of $f(x, y, z)$ over the given region

(a) $f(x, y, z) = y, x^2 + y^2 + z^2 \leq 1, x, y, z \leq 0$

(b) $f(x, y, z) = \rho^{-3}, 2 \leq x^2 + y^2 + z^2 \leq 4$

(c) $f(x, y, z) = 1, x^2 + y^2 + z^2 \leq 4z, z \geq \sqrt{x^2 + y^2}$

5. Compute the volume of a cylinder of height h and radius r using cylindrical coordinates. Compute the volume of a sphere of radius r using spherical coordinates